

Übersetzung und Analyse von Programmen

A stylized, low-poly illustration of a university building with large windows and a red abstract sculpture. In the foreground, there are two green trees and a large yellow flower with a blue center. The sky is blue with a large yellow sun and green geometric shapes.

Vorlesung im Wintersemester 2025/26

Prof. Dr. Stephan Diehl
Informatik, Universität Trier

Zeitplan bis zum Jahresende

11.12.2025	Abstrakte Interpretation (von TRIPLA)	
16.12.2025	Weihnachtsdings + Vortrag Wölwer	Abgabe Übungsblatt „Übersetzung von Java“
18.12.2025		Abgabe Projekt III
		
08.01.2026	Datenflussanalyse	Übungsblatt „Datenflussanalyse“
13.01.2026	Datenflussanalyse von TRIPLA	
15.01.2026	Projekt IV (Kontrollflussanalyse)	Abgabe Übungsblatt „Datenflussanalyse“
20.01.2026	Code Clone Detection	
22.01.2026	Reverse Engineering, Erkennung von Entwurfsmustern	Abgabe Projekt IV
27.01.2026	Gastvortrag: „Programm Analysis and Transformation from a Practitioner’s Point of View“ (Jonas Eckstein, itestra)	Projekt V (Datenflussanalyse)

Datenflussanalyse für TRIPLA



Bildquelle: mit KI erzeugt (Bing, DALL-E)

Formale Beschreibung der **Kontrollflussanalyse**

Gegeben sei die Grammatik G:

```
E → let D in E
   | ID
   | ID ( A )
   | E AOP E
   | ( E )
   | CONST
   | ID = E
   | E ; E
   | if B then E else E
   | while B do { E }
A → E | A , E
D → ID ( V ) { E }
   | D D
V → ID | V , V
B → E | E RELOP E
```

ID: Bezeichner (fangen mit Buchstaben an und bestehen aus Buchstaben und Zahlen)
CONST: ganze Zahlen
AOP: arithm. Operatoren (+, -, *, /)
RELOP: Vergleichsoperatoren (==, !=, <, >)

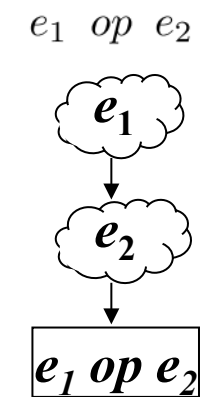
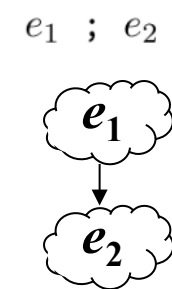
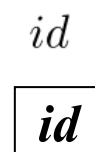
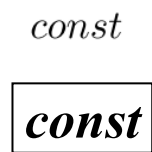
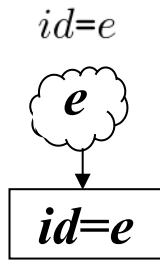
- Wir definieren die Analyse durch Induktion über die Struktur von Ausdrücken:

$$\text{cfg} : L_{G_{TRIPLA}}(E) \times \Psi \longrightarrow \Gamma$$

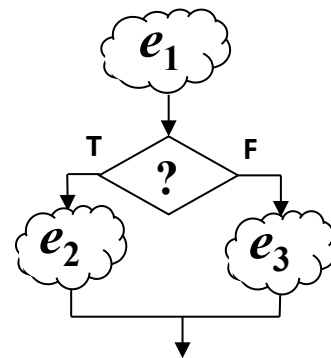
- In der Knotenumgebung werden für Funktionen deren **in** und **out** Knoten gespeichert:

$$\Psi : \textit{Identifizier} \longrightarrow V \times V$$

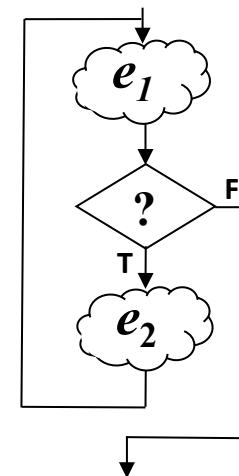
Intraprozedurale Kontrollflussanalyse



if e_1 then e_2 else e_2



while e_1 do { e_2 }

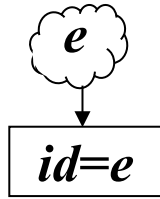


Formale Definition eines Kontrollflussgraphen

A CFG is a tuple (V, E, in, out) where

- V is a set of nodes,
- $E \subseteq V \times L \times V$ a set of edges,
- $L = \{\epsilon, T, F\}$ a set of labels,
- and $in, out \in V$ are start and end nodes.

CFG für einfache Ausdrücke: $\text{cfg}(w) = (V, E, in, out)$, where

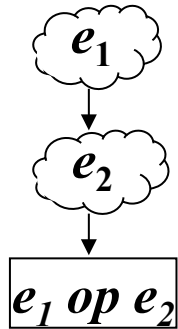


$$\text{if } w = id=e \text{ then } \begin{cases} V = V' \cup \{w\} \\ E = E' \cup \{(out', \epsilon, w)\} \\ in = in' \\ out = w \end{cases}$$

where $(V', E', in', out') = \text{cfg}(e, \psi)$.

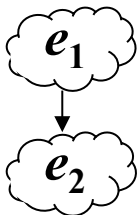


$$\text{if } w = id \text{ or } w = const \text{ then } \begin{cases} V = \{w\} \\ E = \emptyset \\ in = w \\ out = w \end{cases}$$



$$\text{if } w = e_1 \text{ op } e_2 \text{ then } \begin{cases} V = V_1 \cup V_2 \cup \{w\} \\ E = E_1 \cup E_2 \cup \{(out_1, \epsilon, in_2), (out_2, \epsilon, w)\} \\ in = in_1 \\ out = w \end{cases}$$

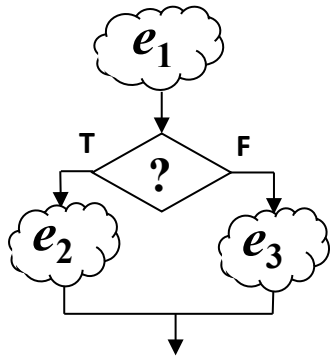
where $(V_1, E_1, in_1, out_1) = \text{cfg}(e_1, \psi)$ and $(V_2, E_2, in_2, out_2) = \text{cfg}(e_2, \psi)$.



$$\text{if } w = e_1 \text{ ; } e_2 \text{ then } \begin{cases} V = V_1 \cup V_2 \\ E = E_1 \cup E_2 \cup \{(out_1, \epsilon, in_2)\} \\ in = in_1 \\ out = out_2 \end{cases}$$

where $(V_1, E_1, in_1, out_1) = \text{cfg}(e_1, \psi)$ and $(V_2, E_2, in_2, out_2) = \text{cfg}(e_2, \psi)$.

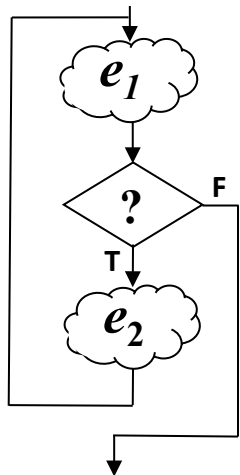
CFGs für IF-THEN-ELSE und WHILE



$$\text{if } w = \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \text{ then}$$

$$\left\{ \begin{array}{l} V = V_1 \cup V_2 \cup V_3 \cup \{diamond, glue\} \\ E = E_1 \cup E_2 \cup E_3 \\ \quad \cup \{(out_1, \epsilon, diamond), (diamond, T, in_2), (diamond, F, in_3), \\ \quad \quad (out_2, \epsilon, glue), (out_3, \epsilon, glue)\} \\ in = in_1 \\ out = glue \end{array} \right.$$

where $(V_i, E_i, in_i, out_i) = \text{cfg}(e_i, \psi)$ and *diamond* and *glue* are two new nodes.

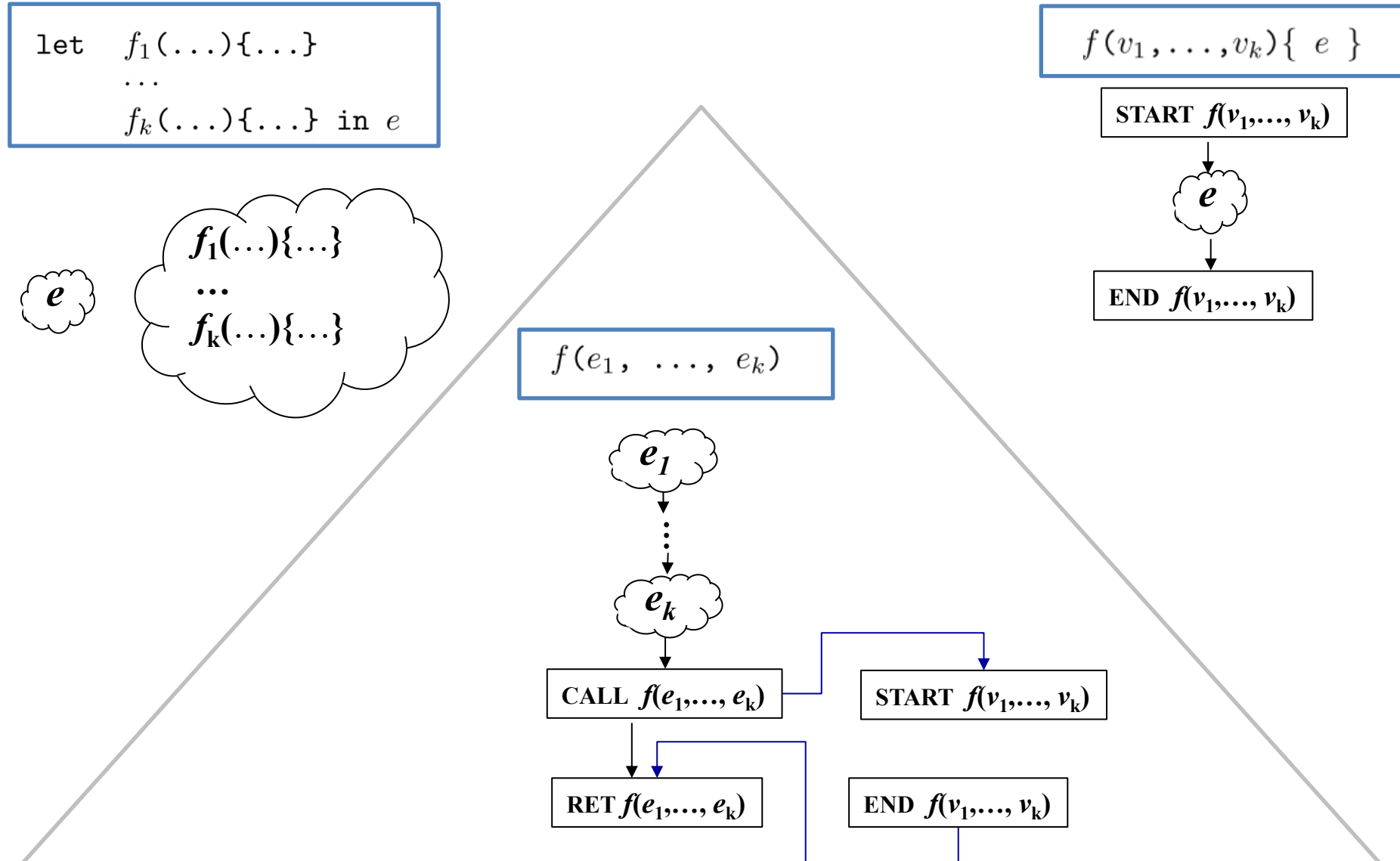


$$\text{if } w = \text{while } e_1 \text{ do } \{ e_2 \}^p \text{ then}$$

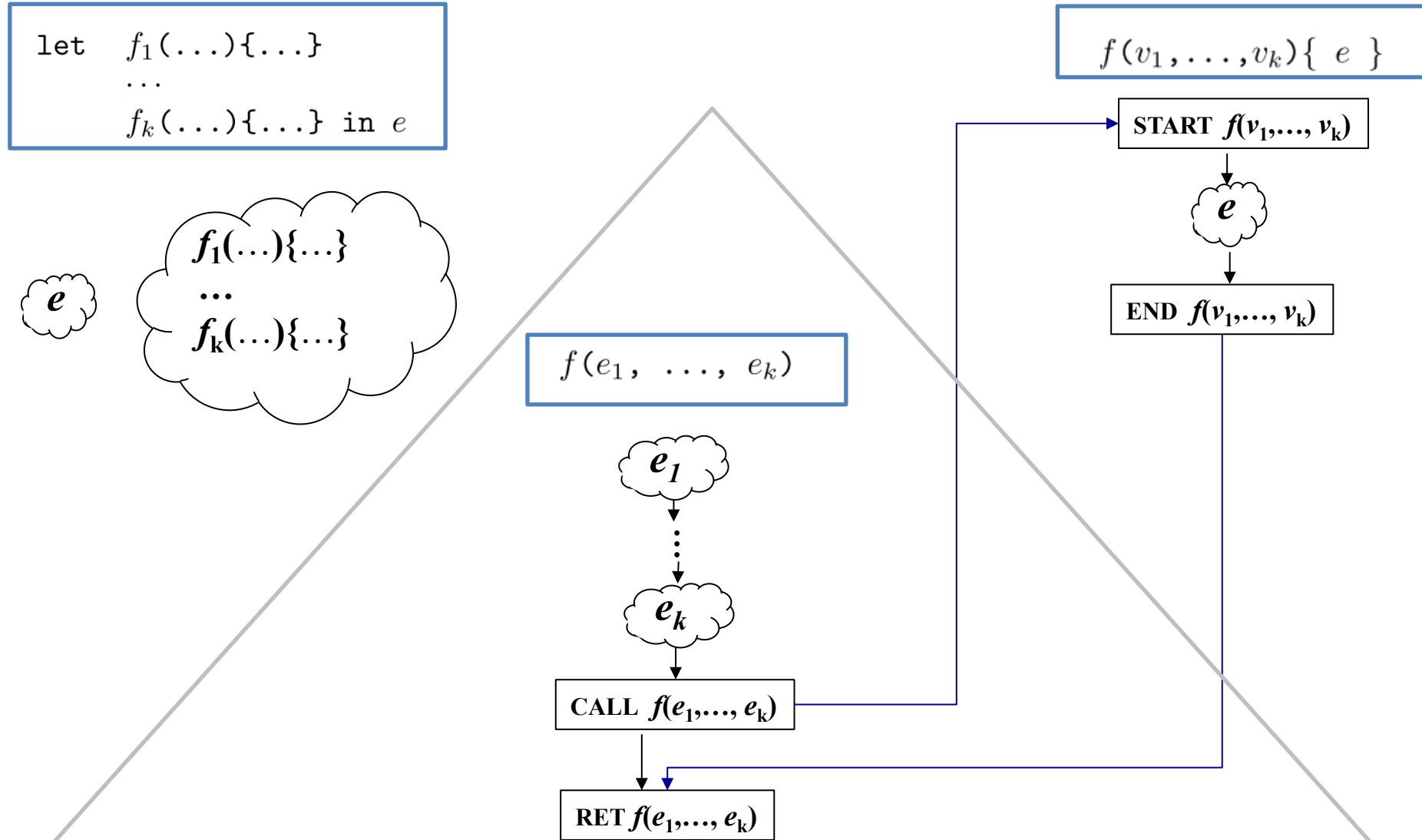
$$\left\{ \begin{array}{l} V = V_1 \cup V_2 \cup \{diamond, glue\} \\ E = E_1 \cup E_2 \cup \{(out_1, \epsilon, diamond), (diamond, T, in_2), \\ \quad (diamond, F, glue), (out_2, \epsilon, in_1)\} \\ in = in_1 \\ out = glue \end{array} \right.$$

where $(V_1, E_1, in_1, out_1) = \text{cfg}(e_1, \psi)$ and $(V_2, E_2, in_2, out_2) = \text{cfg}(e_2, \psi)$ and *diamond* and *glue* are two new nodes.

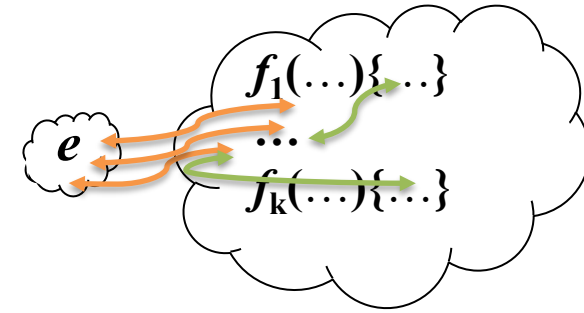
Interprozedurale Kontrollflussanalyse



Interprozedurale Kontrollflussanalyse



CFG für LET-Ausdruck



$\text{cfg}(w, \psi) = (V, E, in, out)$, where

if $w = \text{let } f_1(v_1^1, \dots, v_{n_1}^1)\{e_1\} \cdots f_k(v_1^k, \dots, v_{n_k}^k)\{e_k\} \text{ in } e$ then

$$\begin{cases} V = V' \cup \bigcup V_i \\ E = E' \cup \bigcup E_i \\ in = in' \\ out = out' \end{cases}$$

Funktioniert auch für
verschachtelte LETs

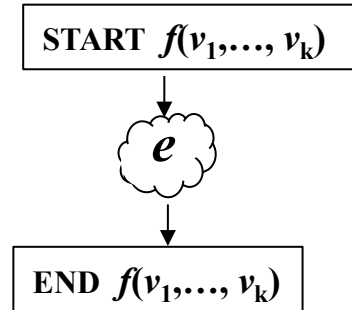
where $\psi' = \psi[(in_1, out_1)/f_1] \cdots [(in_k, out_k)/f_k]$

with $in_i = (\text{START}, f_i(v_1^i, \dots, v_{n_i}^i))$, $out_i = (\text{END}, f_i(v_1^i, \dots, v_{n_i}^i))$,

and $(V', E', in', out') = \text{cfg}(e, \psi')$

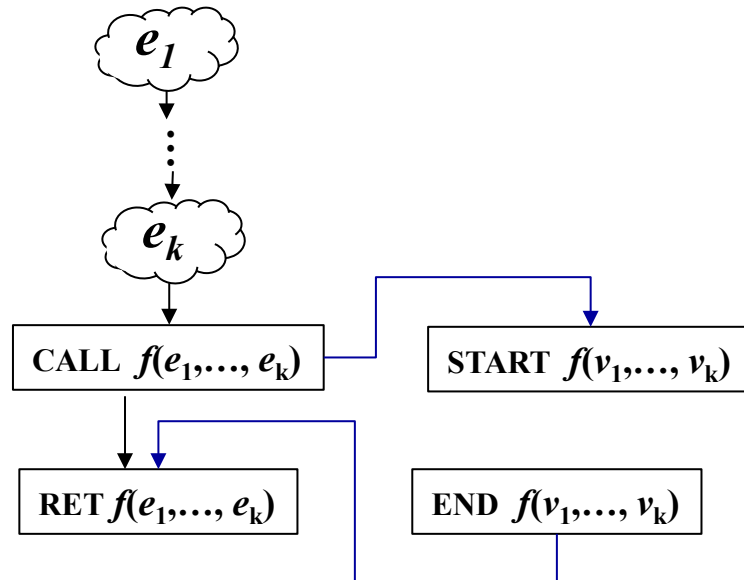
and $(V_i, E_i, in_i, out_i) = \text{cfg}(f_i(v_1^i, \dots, v_{n_i}^i)\{e_i\}, \psi')$

CFGs für Methodendefinition und -aufruf



$$\text{if } w = f(v_1, \dots, v_k) \{ e \} \text{ then} \quad \begin{cases} V = V' \cup \{ in_f, out_f \} \\ E = E' \cup \{ (in_f, \epsilon, in'), (out', \epsilon, out_f) \} \\ in = in_f \\ out = out_f \end{cases}$$

where $(V', E', in', out') = \text{cfg}(e, \psi)$ and $(in_f, out_f) = \psi(f)$.



$$\text{if } w = f(e_1, \dots, e_k) \text{ then} \quad \begin{cases} V = \{ in_f, out_f, out \} \cup \bigcup V_i \\ E = \{ (out_1, \epsilon, in_2), \dots, (out_{k-1}, \epsilon, in_k), \\ \quad (out_k, \epsilon, call), (call, \epsilon, in_f), (out_f, \epsilon, ret), \\ \quad (call, \epsilon, ret) \} \cup \bigcup E_i \\ in = in_1 \\ out = ret \end{cases}$$

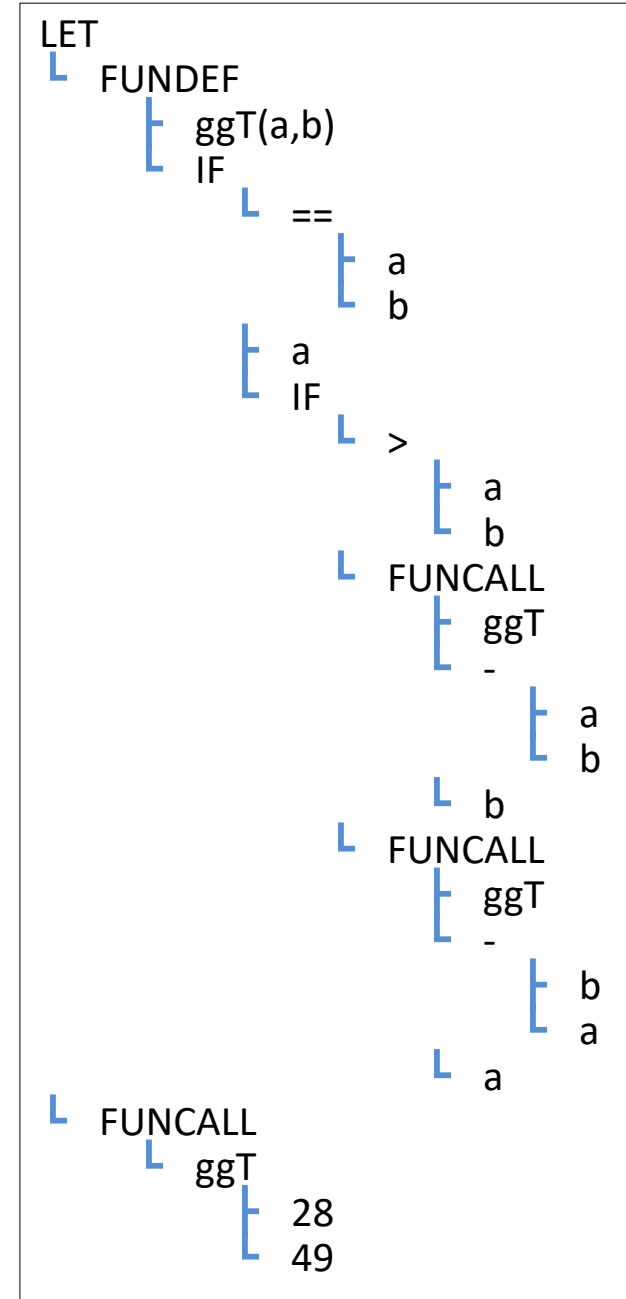
where $(V_i, E_i, in_i, out_i) = \text{cfg}(e_i, \psi)$,
 $(in_f, out_f) = \psi(f)$ and $call = (\text{CALL}, w)$, $ret = (\text{RET}, w)$.

```
let ggT(a,b) {  
  if (a==b) then  
    a  
  else  
    if (a>b) then  
      ggT(a-b,b)  
    else  
      ggT (b-a,a) }  
in ggT(28, 49)
```

```

let ggT(a,b) { if (a==b) then a
                else if (a>b) then ggT(a-b,b)
                else ggT (b-a,a) }
in ggT(28, 49)

```

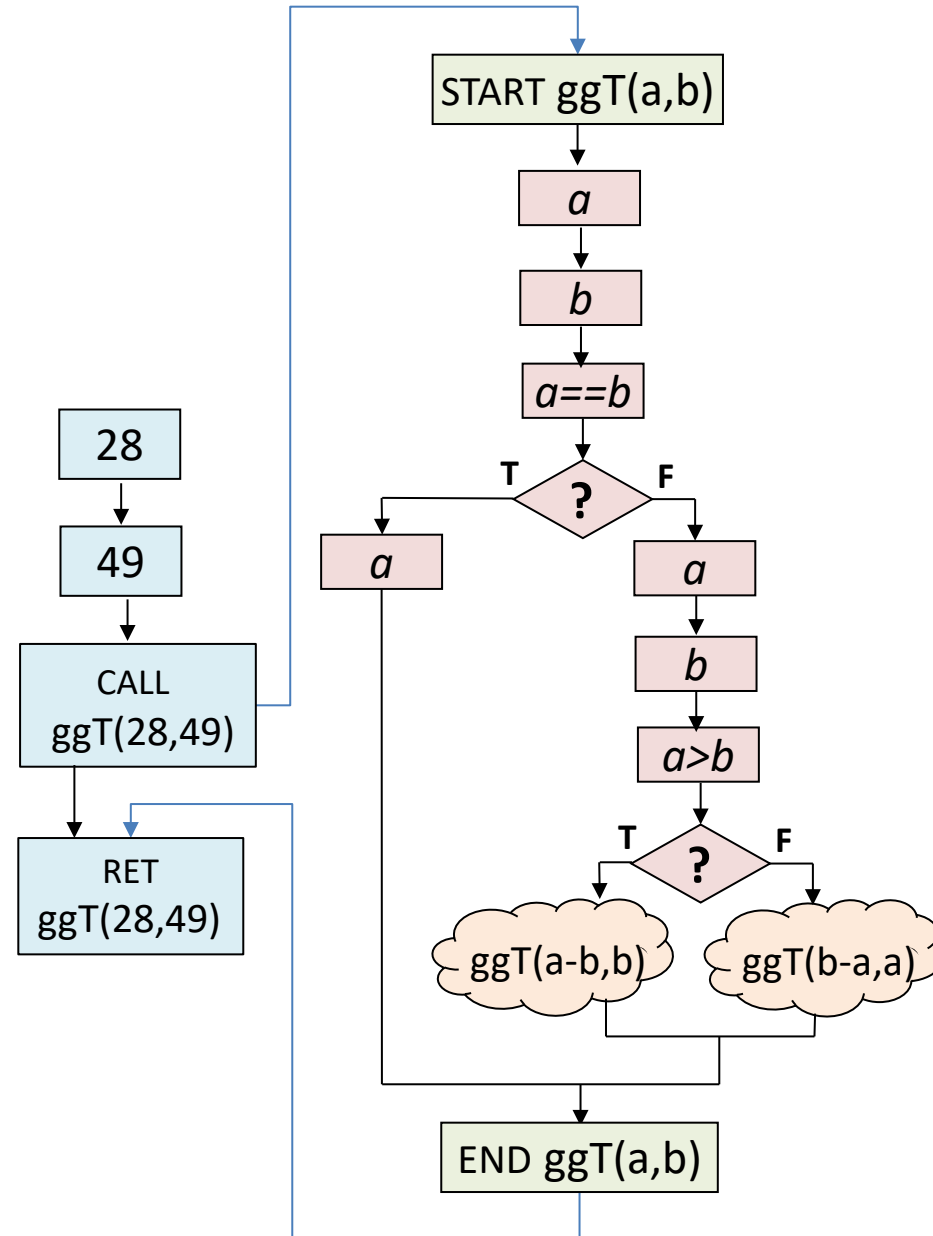


TRIPLA Quellcode

3

```
let ggT(a,b) { if (a==b) then a  
               else if (a>b) then ggT(a-b,b)  
               else ggT (b-a,a) }  
in ggT(28, 49)
```

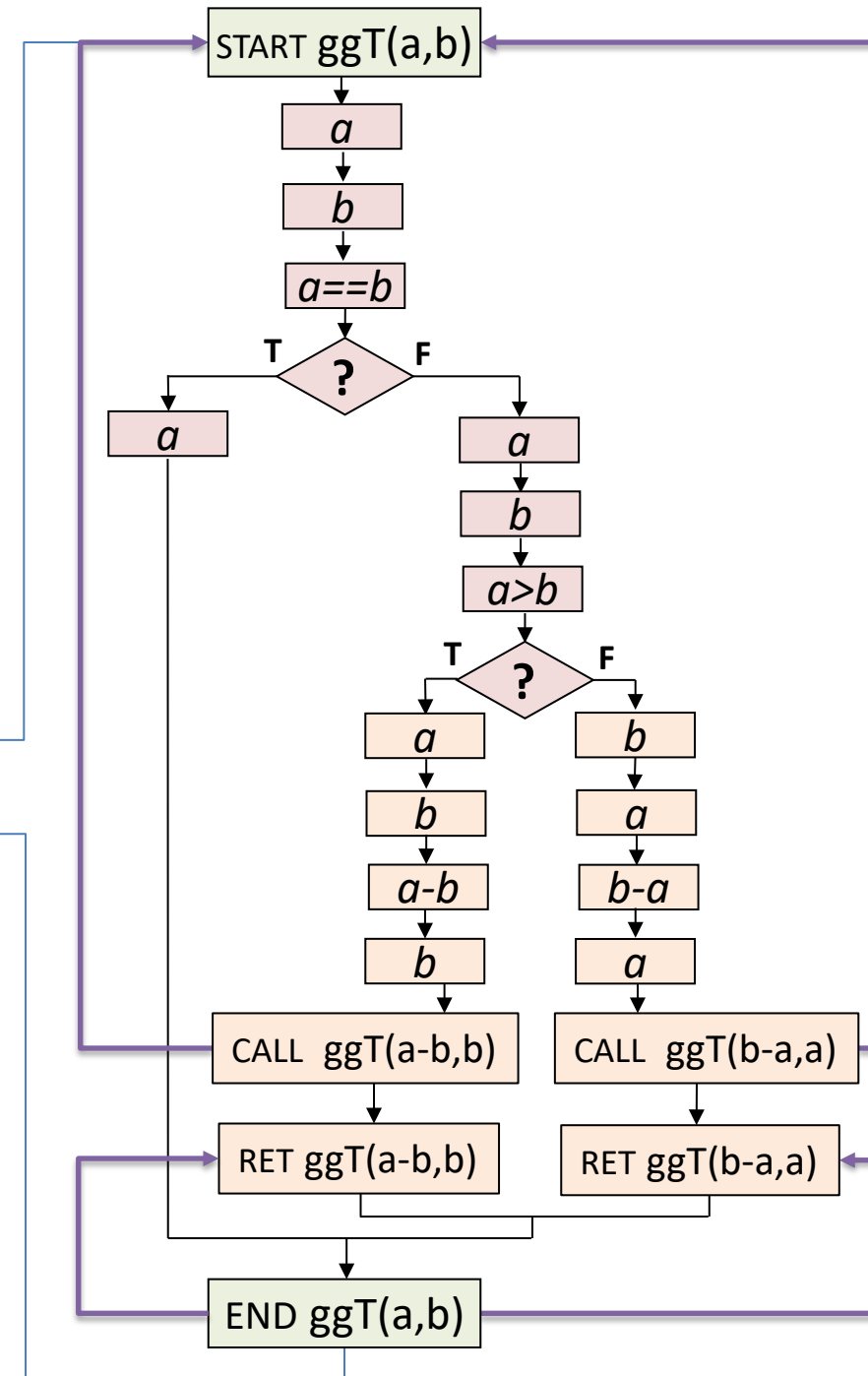
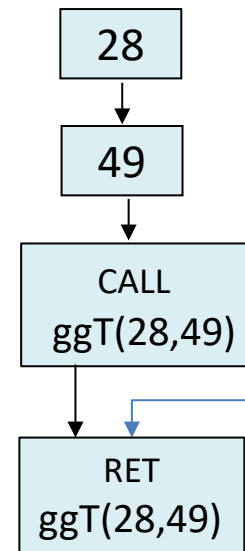
endrekursiv



TRIPLA Quellkode

```
let ggT(a,b) { if (a==b) then a
               else if (a>b) then ggT(a-b,b)
               else ggT (b-a,a) }
in ggT(28, 49)
```

4

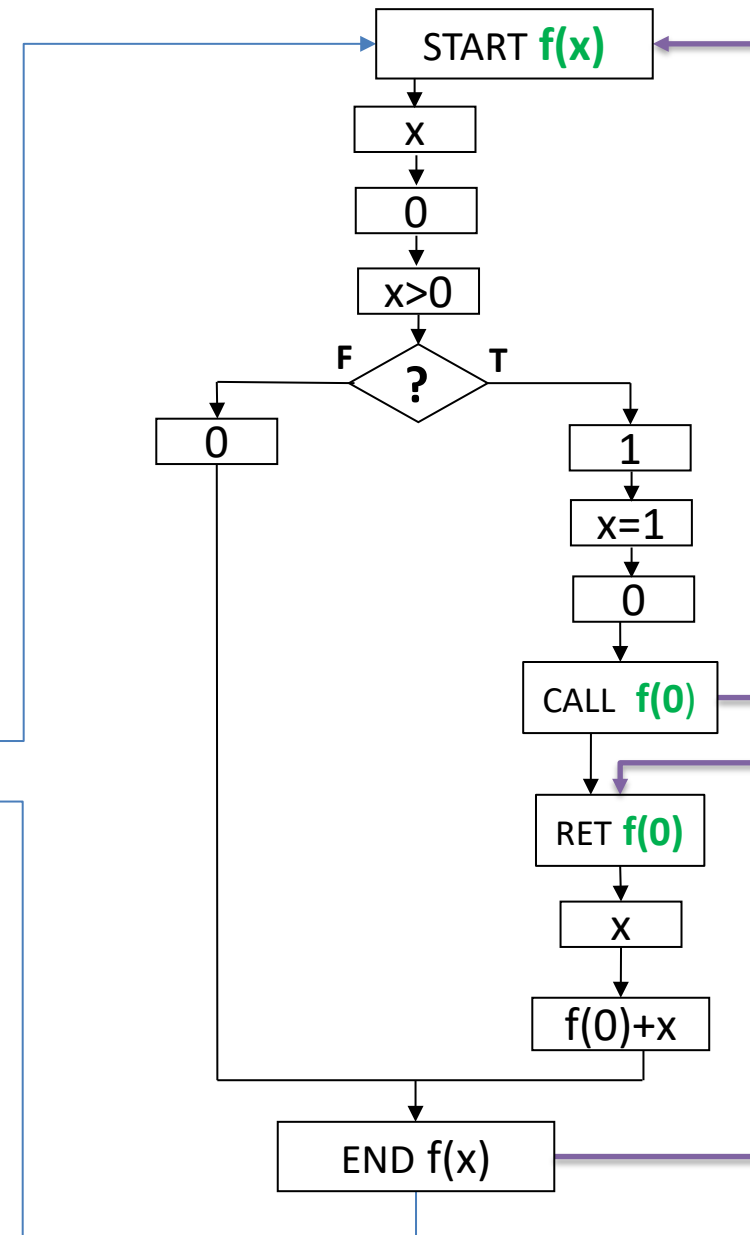
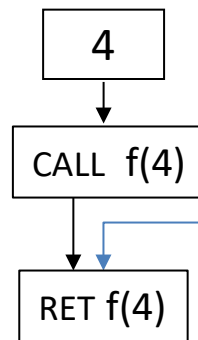


Rekursion

TRIPLA Quellcode

```
let f(x) { if (x>0) then  
           x=1; f(0)+x  
           else 0 }  
in f(4)
```

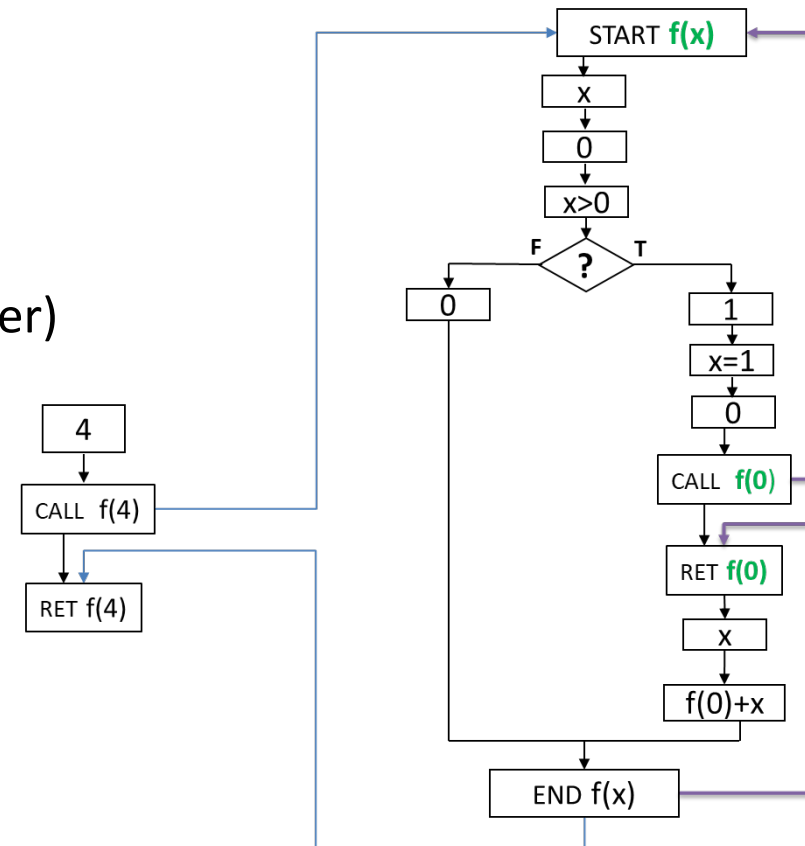
rekursiv, aber nicht endrekursiv



Interprozedurale Datenflussanalyse

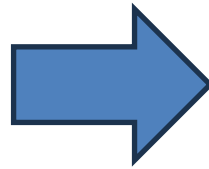
- Knotentypen
 - x (lesender Zugriff auf x)
 - c (Konstante)
 - e1 op e2 (binärer Ausdruck)
 - x=e (Zuweisung)
 - <?> („if true“, Bedingung steht in Vorgänger)
 - CALL
 - RET
 - START
 - END

Implementierungshinweis:
Statt Mengen von Kanten verwendet man besser Pointer.



Konstantenpropagation: Beispiel

- $a=0$
- $b=1$
- $c=2$
- if ($d>0$) $a=5$ else $b=a$
- $d=a*b+c$



- $a=0$
- $b=1$
- $c=2$
- if ($d>0$) $a=5$ else $b=0$
- $d=a*b+2$

Vorwärtsflussproblem: Konstantenpropagation

$$\begin{aligned}
 IN(in) &= \emptyset & \mathcal{E} &= L_G(ID) \times L_G(CONST) \\
 OUT(in) &= \emptyset \\
 \text{for all } v \in (V - \{in\}) \text{ do} \\
 & IN(v) = \mathcal{E} \\
 & OUT(v) = (IN(v) - KILL(v)) \cup GEN(v) \\
 \text{while there are changes do} \\
 & \text{for all } v \in (V - \{in\}) \text{ do} \\
 & \quad IN(v) = \bigcap_{(p,l,v) \in E} OUT(p) \\
 & \quad OUT(v) = (IN(v) - KILL(v)) \cup GEN(v)
 \end{aligned}$$

Implementierungshinweis:

$\mathcal{E}$ nicht erzeugen, sondern als speziellen Wert darstellen und als Sonderfall bei Mengenoperationen behandeln.

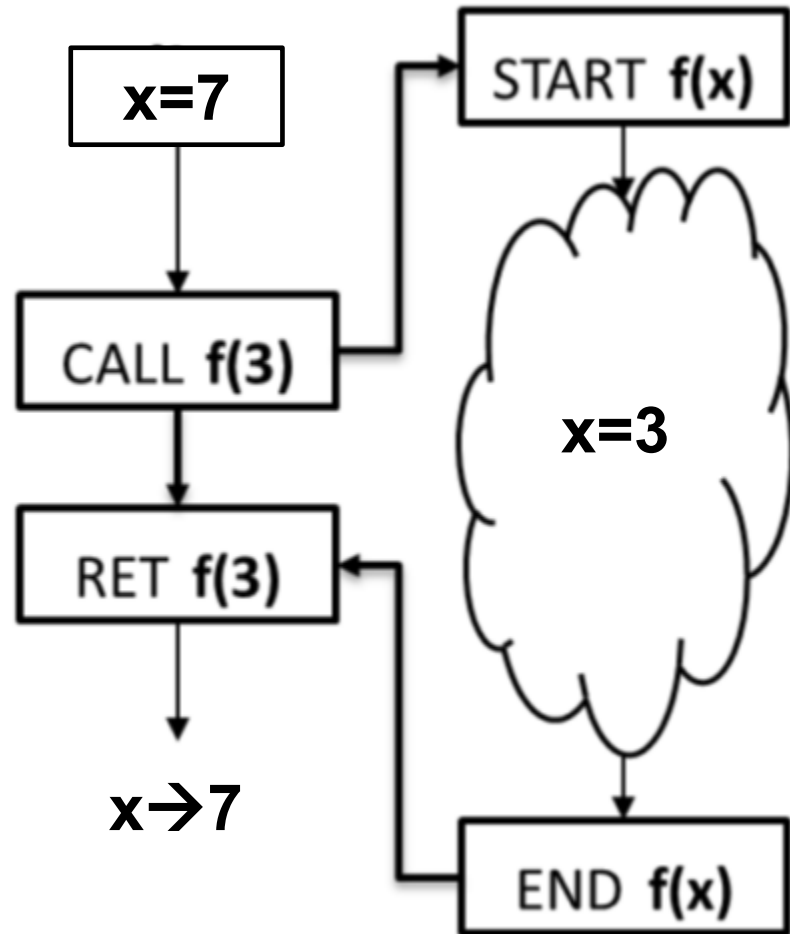
$$OUT(v) := \{(id, c) \mid (id, c) \in IN(v) \text{ and } id \notin KILLVARS(v)\} \cup GEN(v)$$

$$KILLVARS(v) = \begin{cases} \{id\} & \text{if } v = id=e \\ \{v_1, \dots, v_k\} & \text{if } v = (START, f(v_1, \dots, v_k)) \\ \{v_1, \dots, v_k\} & \text{if } v = (END, f(v_1, \dots, v_k)) \\ \emptyset & \text{otherwise} \end{cases}$$

$$KILL(v) = \{(id, c) \mid id \in KILLVARS(v) \text{ and } c \text{ is a constant}\}$$

$$GEN(v) = \begin{cases} \{(id, c)\} & \text{if } v = id=c \\ \emptyset & \text{otherwise} \end{cases}$$

Analog zu
Erreichbaren
Ausdrücken



Rückwärtsflussproblem: Lebendige Variablen

$$KILLVARS(v) = \begin{cases} \{id\} & \text{if } v = id=e \\ \{v_1, \dots, v_k\} & \text{if } v = (\text{START}, f(v_1, \dots, v_k)) \\ \{v_1, \dots, v_k\} & \text{if } v = (\text{END}, f(v_1, \dots, v_k)) \\ \emptyset & \text{otherwise} \end{cases}$$

```
for all  $v \in V$  do
   $IN(v) := GEN(v)$ 
while there are changes do
  for all  $v \in V$  do
    if  $v == (\text{CALL}, w)$  then
       $\exists! ret \in \text{succ}(v)$  with  $ret = (\text{RET}, w)$ 
       $\exists! start \in \text{succ}(v)$  with  $start = (\text{START}, w')$ 
       $\exists! end \in \text{pred}(ret)$  with  $end = (\text{END}, w')$ 
       $\Delta := (OUT(end) - OUT(start)) - KILL(start)$ 
       $IN(v) := (OUT(ret) \cup IN(start)) - \Delta$ 
    else
       $OUT(v) := \bigcup_{s \in \text{succ}(v)} IN(s)$ 
       $IN(v) := (OUT(v) - KILL(v)) \cup GEN(v)$ 
```

$$KILL(v) = KILLVARS(v)$$
$$GEN(v) = \begin{cases} \{id\} & \text{if } v = id \\ \emptyset & \text{otherwise} \end{cases}$$

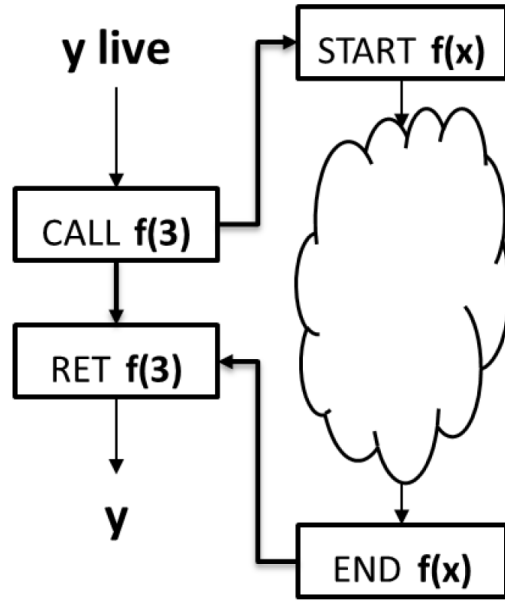
Hinweis:

In der vorherigen Vorlesung wurde USE und DEF statt GEN und KILL verwendet, um die Mengen zu benennen..

Interprozedurale DFA: Lebendige Variablen

1. Fall: Äußere Variable nicht durch Parameter überdeckt

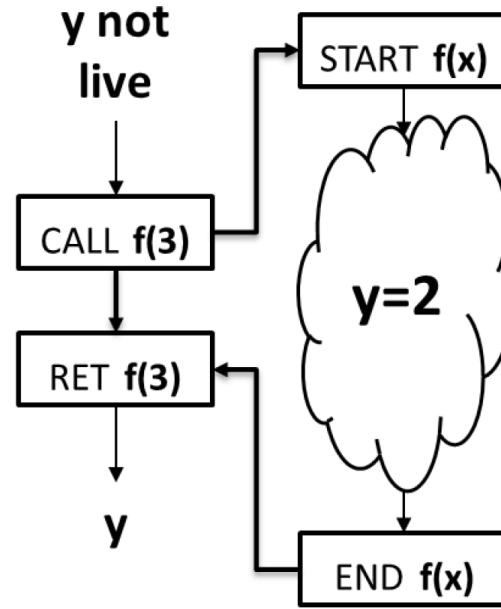
Rückwärtsflussanalyse



(a) The variable y is read after the function call, but neither written nor read in $f()$.

$$\Delta = (\{y\} - \{y\}) - \{x\} = \emptyset$$

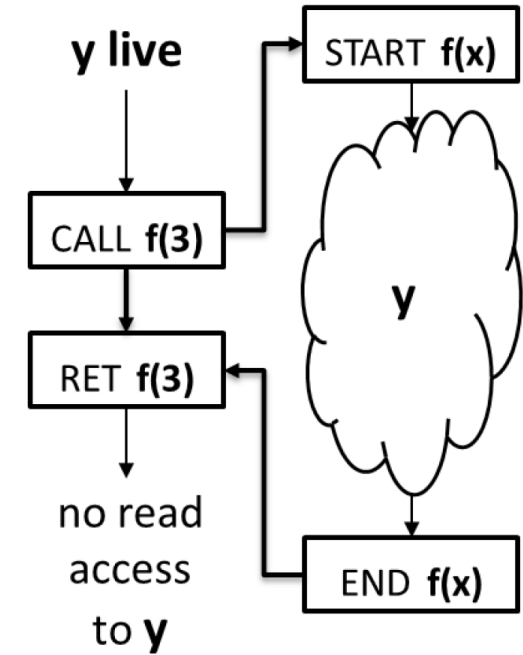
$$IN = (\{y\} \cup \emptyset) - \emptyset = \{y\}$$



(b) The variable y is read after the function call, but written along all paths in $f()$.

$$\Delta = (\{y\} - \emptyset) - \{x\} = \{y\}$$

$$IN = (\{y\} \cup \{y\}) - \{y\} = \emptyset$$



(c) There is at least one path along which the variable y is not written before it is read in $f()$.

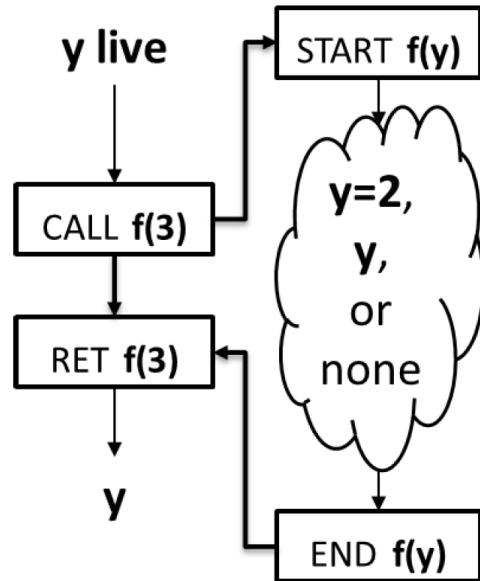
$$\Delta = (\emptyset - \{y\}) - \{x\} = \emptyset$$

$$IN = (\emptyset \cup \{y\}) - \emptyset = \{y\}$$

Interprozedurale DFA: Lebendige Variablen

2. Fall: Äußere Variable durch gleichnamigen Parameter überdeckt

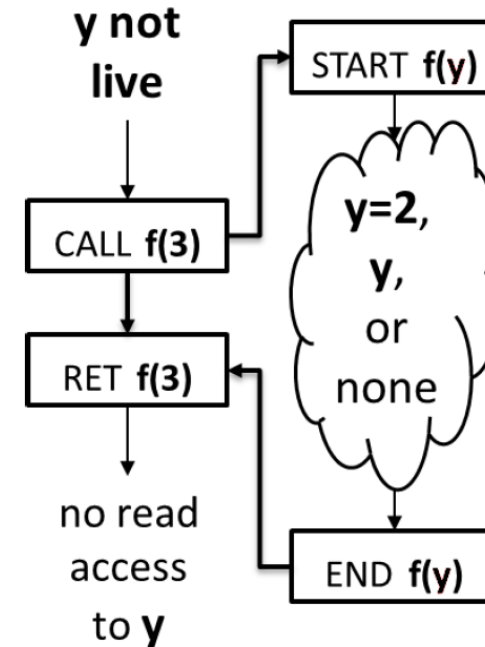
Rückwärtsflussanalyse



(a) The variable y is read after the function call. All occurrences of y in $f()$ refer to the parameter with the same name, not to the outer variable y .

$$\Delta = (\{y\} - M) - \{y\} = \emptyset$$

$$IN = (\{y\} \cup \emptyset) - \emptyset = \{y\}$$



(b) The variable y is not read after the function call. All occurrences of y in $f()$ refer to the parameter with the same name, not to the outer variable y .

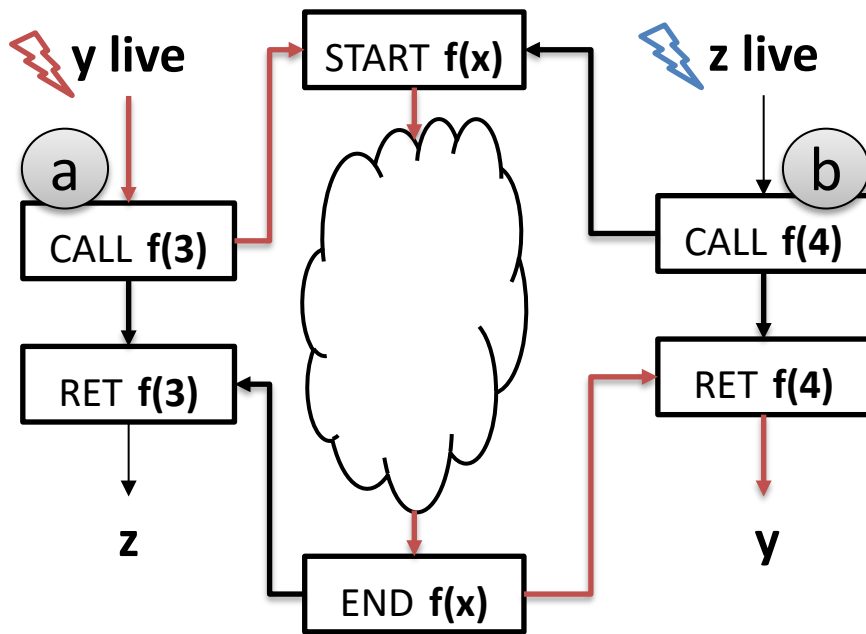
$$\Delta = (\emptyset - M) - \{y\} = \emptyset$$

$$IN = (\emptyset \cup \emptyset) - \emptyset = \emptyset$$

For the computation: $OUT(start) = M$ where $M = \{y\}$ if y is read and not written along at least one path in f and $M = \emptyset$ otherwise.

Interprozedurale DFA: Lebendige Variablen

Problem: Propagation entlang ungültiger Pfade

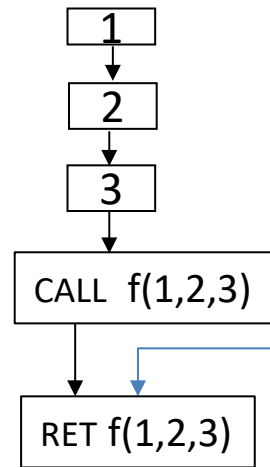


$$\Delta = (\{y, z\} - \{y, z\}) - \{x\} = \emptyset$$

$$\text{a) } IN = (\{z\} \cup \{y, z\}) - \emptyset = \{y, z\}$$

$$\text{b) } IN = (\{y\} \cup \{y, z\}) - \emptyset = \{y, z\}$$

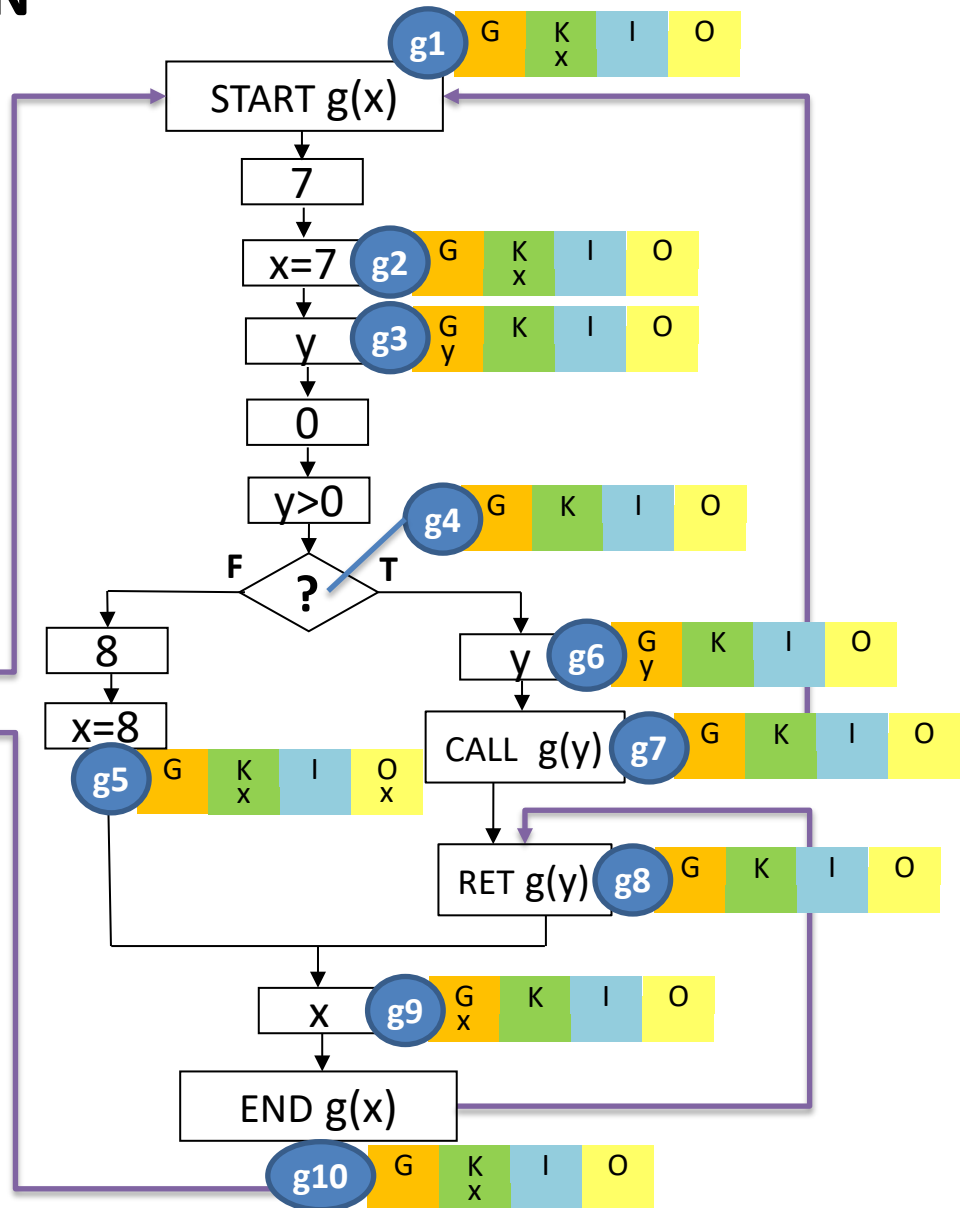
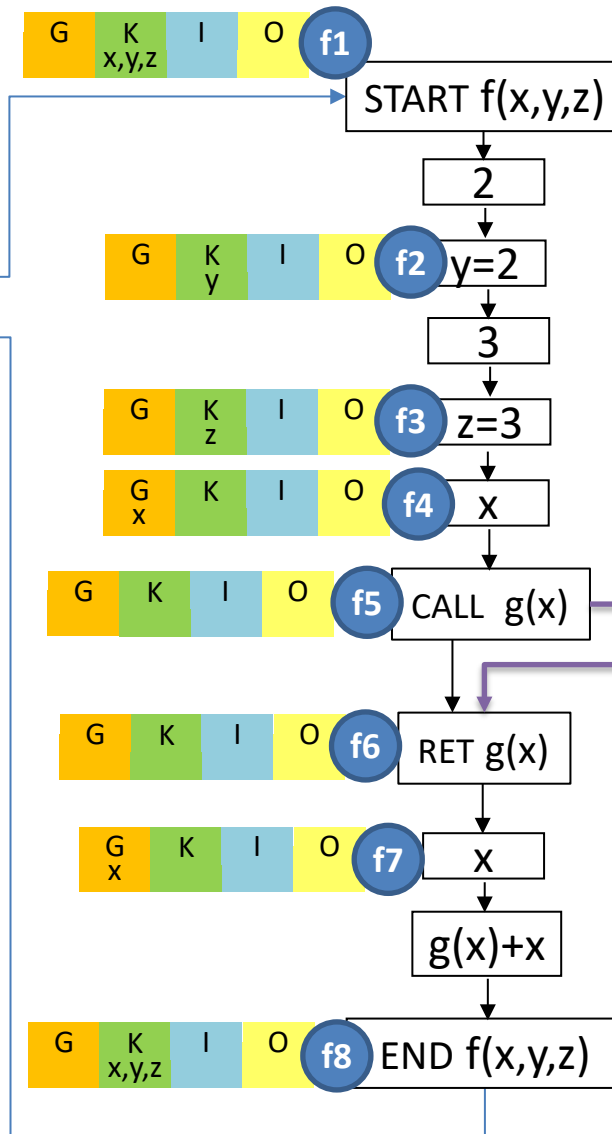
Lebendige Variablen: KILL und GEN



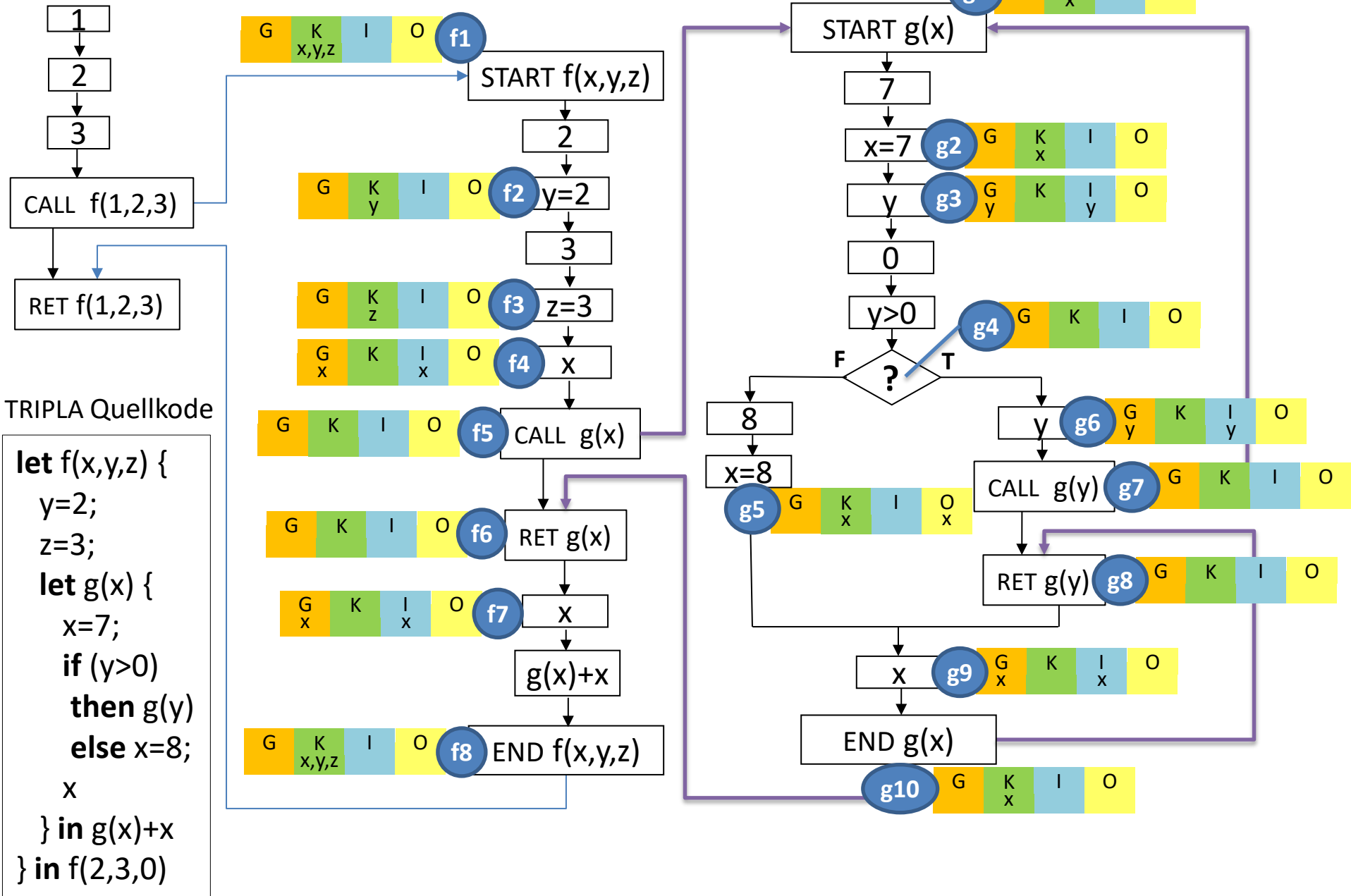
TRIPLA Quellcode

```

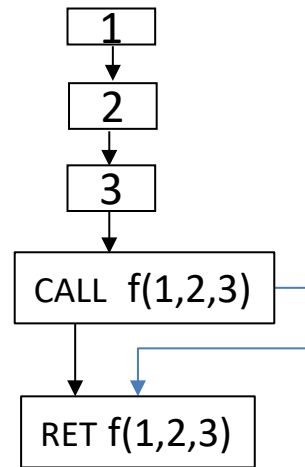
let f(x,y,z) {
  y=2;
  z=3;
  let g(x) {
    x=7;
    if (y>0)
    then g(y)
    else x=8;
    x
  } in g(x)+x
} in f(2,3,0)
  
```



Lebendige Variablen: Intialisierung IN=GEN



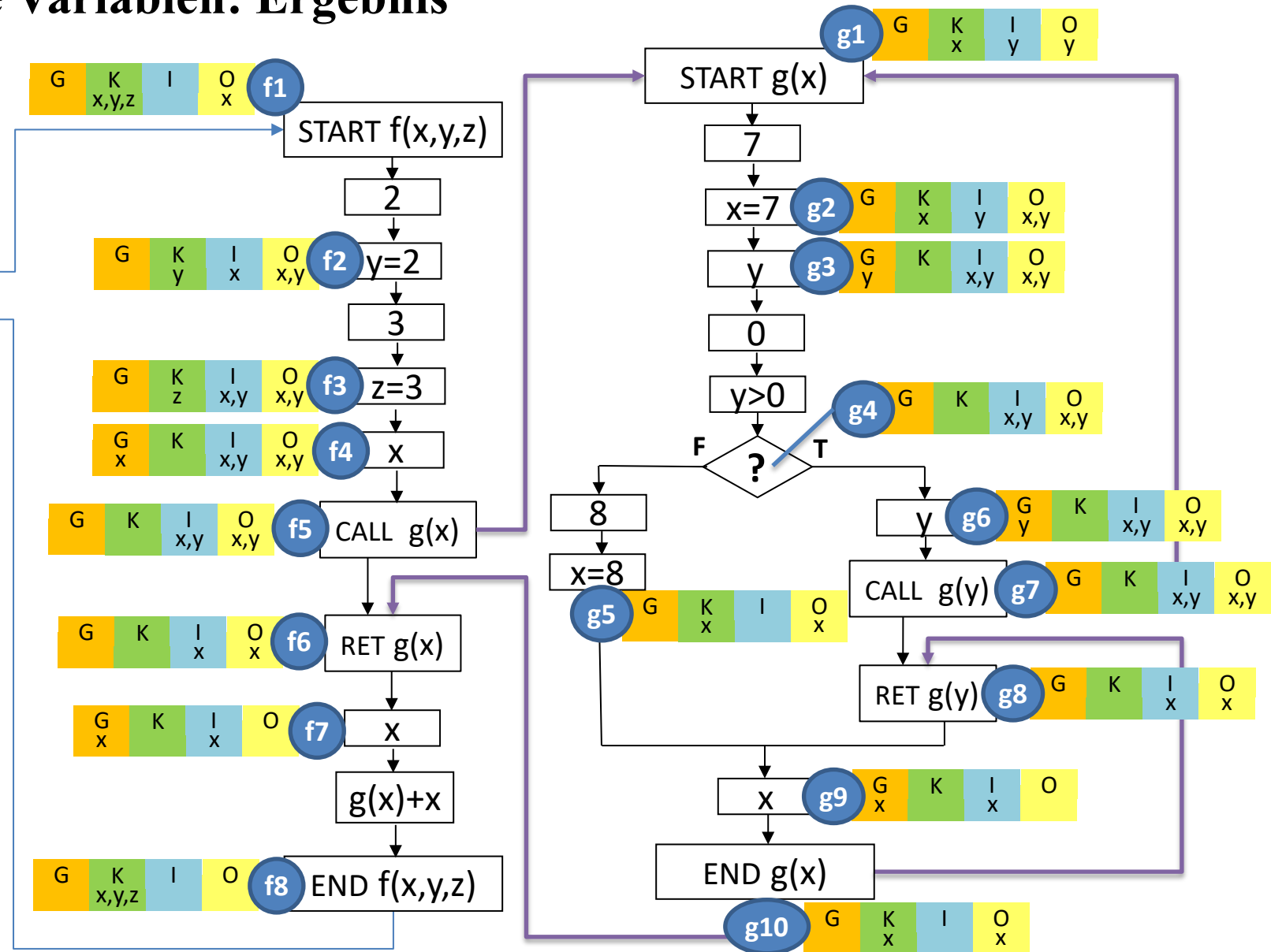
Lebendige Variablen: Ergebnis



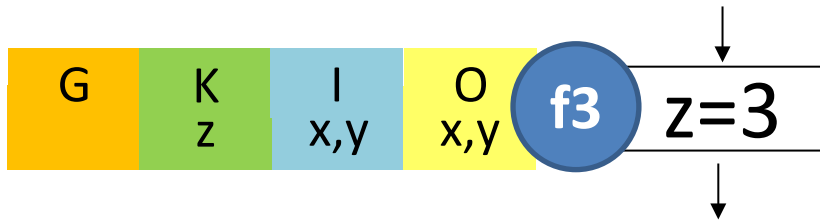
TRIPLA Quellcode

```

let f(x,y,z) {
  y=2;
  z=3;
  let g(x) {
    x=7;
    if (y>0)
    then g(y)
    else x=8;
    x
  } in g(x)+x
} in f(2,3,0)
  
```



Programmoptimierung



- z ist hier nicht lebendig, d.h. es gibt keine zukünftige Verwendung von z . Daher kann auf die Zuweisung verzichtet werden, im Allgemeinen jedoch wegen möglicher Seiteneffekte nicht auf die Auswertung der rechten Seite der Zuweisung.
- Im Programmcode kann $id = e$ durch e ersetzt werden, wenn für den entsprechenden Knoten v im Kontrollflussgraphen $id \notin OUT(v)$ gilt.

Rückwärtsflussproblem: Reached Uses (Erreichte Verwendungen)

$$KILL(v) = \begin{cases} \{(p', id) \mid p' \in P\} & \text{if } v = id = e^P \\ \{(p', id_1), \dots, (p', id_k) \mid p' \in P\} & \text{if } v = (START, f(id_1, \dots, id_k)^P) \\ \{(p', id_1), \dots, (p', id_k) \mid p' \in P\} & \text{if } v = (END, f(id_1, \dots, id_k)^P) \\ \emptyset & \text{otherwise} \end{cases}$$

$$GEN(v) = \begin{cases} \{(p, id)\} & \text{if } v = id^P \\ \emptyset & \text{otherwise} \end{cases}$$

```

for all  $v \in V$  do
     $IN(v) := GEN(v)$ 
while there are changes do
    for all  $v \in V$  do
        if  $v == (CALL, w)$  then
             $\exists! ret \in succ(v)$  with  $ret = (RET, w)$ 
             $\exists! start \in succ(v)$  with  $start = (START, w')$ 
             $\exists! end \in pred(ret)$  with  $end = (END, w')$ 
             $\Delta := (OUT(end) - OUT(start)) - KILL(start)$ 
             $IN(v) := (OUT(ret) \cup IN(start)) - \Delta$ 
        else
             $OUT(v) := \bigcup_{s \in succ(v)} IN(s)$ 
             $IN(v) := (OUT(v) - KILL(v)) \cup GEN(v)$ 
    
```

**Analog zu
Lebendige
Variablen**

Rückwärtsflussproblem: Reached Uses (Erreichte Verwendungen)

$$KILL(v) = \begin{cases} \{id\} & \text{if } v = id = e^p \\ \{id_1, \dots, id_k\} & \text{if } v = (START, f(id_1, \dots, id_k)^p) \\ \{id_1, \dots, id_k\} & \text{if } v = (END, f(id_1, \dots, id_k)^p) \\ \emptyset & \text{otherwise} \end{cases}$$

$$GEN(v) = \begin{cases} \{(p, id)\} & \text{if } v = id^p \\ \emptyset & \text{otherwise} \end{cases}$$

for all $v \in V$ do
 $IN(v) := GEN(v)$

while there are changes do

for all $v \in V$ do

if $v == (CALL, w)$ then

$\exists! ret \in succ(v)$ with $ret = (RET, w)$

$\exists! start \in succ(v)$ with $start = (START, w')$

$\exists! end \in pred(ret)$ with $end = (END, w')$

$\Delta := (OUT(end) - OUT(start)) \ominus KILL(start)$

$IN(v) := (OUT(ret) \cup IN(start)) - \Delta$

else

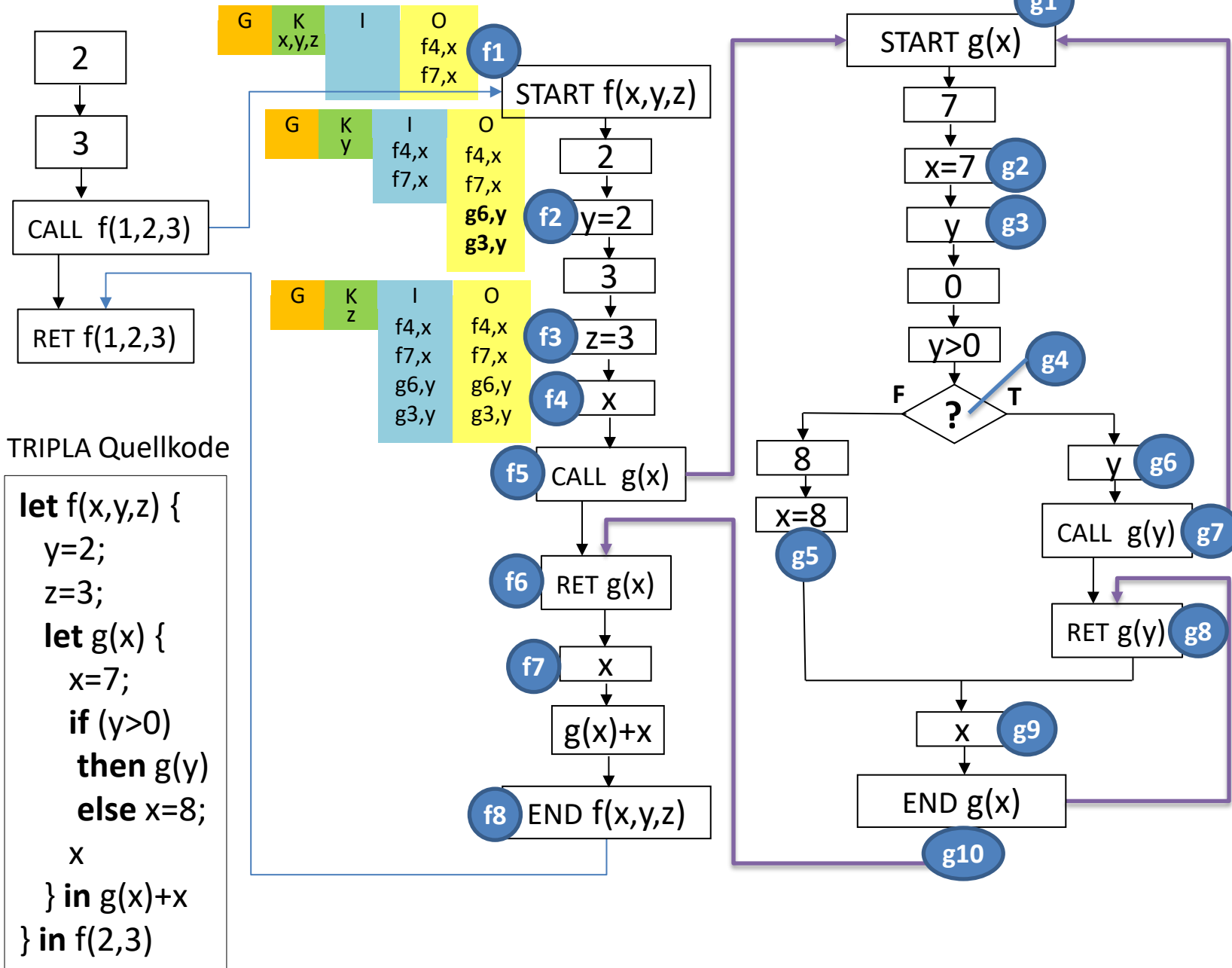
$OUT(v) := \bigcup_{s \in succ(v)} IN(s)$

$IN(v) := (OUT(v) \ominus KILL(v)) \cup GEN(v)$

$$M \ominus K = \{(p, id) \mid (p, id) \in M \text{ and } id \notin K\}$$

**Analog zu
Lebendige
Variablen**

Reached Uses



if $v == (\text{CALL}, w)$ then
 $\exists !ret \in \text{succ}(v)$ with $ret = (\text{RET}, w)$
 $\exists !start \in \text{succ}(v)$ with $start = (\text{START}, w')$
 $\exists !end \in \text{pred}(ret)$ with $end = (\text{END}, w')$
 $\Delta := (\text{OUT}(end) - \text{OUT}(start)) \ominus \text{KILL}(start)$
 $IN(v) := (\text{OUT}(ret) \cup IN(start)) - \Delta$
 else
 $OUT(v) := \bigcup_{s \in \text{succ}(v)} IN(s)$
 $IN(v) := (\text{OUT}(v) \ominus \text{KILL}(v)) \cup \text{GEN}(v)$

Nodes (in order of visit)	GEN	KILL	$OUT_0 = \{\}$ $OUT_i = \text{Union of IN of successors}$	$IN_0 = GEN$ $IN_i = (OUT - KILL) + GEN$			
			OUT_1	IN_1	OUT_2	IN_2	
f8							
f7	(f7,x)			(f7,x)			
f6			(f7,x)	(f7,x)			
g10		(_,x)	(f7,x)		(f7,x),(g9,x)		no changes to propagate
g9	(g9,x)			(g9,x)			
g8			(g9,x)	(g9,x)			
g7			(g9,x)	(g9,x)	(g9,x),(g6,y),(g3,y)	(g9,x),(g6,y),(g3,y)	
g6	(g6,y)		(g9,x)	(g9,x),(g6,y)	(g9,x),(g6,y),(g3,y)	(g9,x),(g6,y),(g3,y)	
g5		(_,x)	(g9,x)				
g4			(g9,x),(g6,y)	(g9,x),(g6,y)	(g9,x),(g6,y),(g3,y)	(g9,x),(g6,y),(g3,y)	
g3	(g3,y)		(g9,x),(g6,y)	(g9,x),(g6,y),(g3,y)	(g9,x),(g6,y),(g3,y)	(g9,x),(g6,y),(g3,y)	no change to propagate
g2		(_,x)	(g9,x),(g6,y),(g3,y)	(g6,y),(g3,y)			
g1		(_,x)	(g6,y),(g3,y)	(g6,y),(g3,y)			
f5			(f7,x),(g6,y),(g3,y)	(f7,x),(g6,y),(g3,y)	(f7,x),(g6,y),(g3,y)		
f4	(f4,x)		(f7,x),(g6,y),(g3,y)	(f4,x),(f7,x),(g6,y),(g3,y)			
f3		(_,z)	(f4,x),(f7,x),(g6,y),(g3,y)	(f4,x),(f7,x),(g6,y),(g3,y)			
f2		(_,y)	(f4,x),(f7,x),(g6,y),(g3,y)	(f4,x),(f7,x)			
f1		(_,x),(_,y),(_,z)	(f4,x),(f7,x)				

Tabelle nicht auf dem neusten Stand !!!

2nd Iteration: Where did the IN-value of a successor change? => When was a node visited before one of its predecessors? => Where are cycles? => Where are backward edges? => g10->g8 and g7->g1

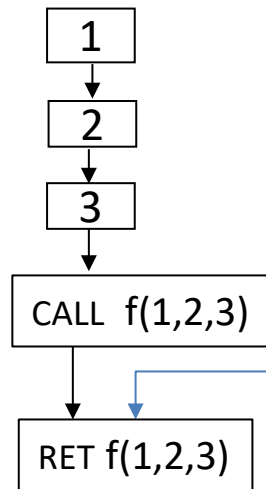
Datenflussgraph

Mit Hilfe der erreichbaren Verwendungen (Reached Uses) kann der Datenflussgraph konstruiert werden:

$$G_D = (V, E_D)$$

$$E_D = \{(v, id^p) \mid v \in V, (p, id) \in OUT(v) \\ \text{and } v = (START, f(id_1, \dots, id_k)^p) \text{ where } \exists i : id_i = id \\ \text{or } v = id = e^p\} \cup E$$

Es wird eine Datenflusskante von jeder Zuweisung $x=e$ zu den Verwendungen (lesenden Zugriffen) der Variablen x erzeugt, sowie vom Eingangsknoten des CFGs jeder Funktion zu den Verwendungen ihrer Parameter.



TRIPLA Quellkode

```

let f(x,y,z) {
  y=2;
  z=3;
  let g(x) {
    x=7;
    if (y>0)
      then g(y)
    else x=8;
    x
  } in g(x)+x
} in f(2,3)

```

